DERIVED GEOMETRY LEARNING SEMINAR II

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In this semester, we continue to learn some basic derived algebraic geometry. This time, we will focus on higher category theory, which plays an central role in the more systematical approach (a la Lurie) to derived algebraic geometry. To begin, we will start by learning basic notations of infinity categories via the model of quasi-categories, following Lurie. Then we will move on to the basic theory of presentable ∞-categories, and ∞-topoi. If time permits, we will also discuss some stable ∞-categories. The main reference is Lurie's HTT and HA [4, 5], as well as Cisinski [1] and Land [3].

Lecture 1. Introduction

Given a quick introduction to the higher category theory and introduce the notation of quasi-category as our main model of $(\infty, 1)$-category (denoted as ∞-category below). The main references are [4, Chapter 1] and [3, Section 1.1-1.2]

Lecture 2. Simplicial model category, homotopy category and examples

A quick review of simplicial model category and then construct the corresponding ∞-category. Construct the ∞-category of spaces (Kan complexes) from the category of simplicial sets, which plays a fundamental role in ∞-categories (similar as the category of sets). Construct the homotopy category of an ∞-category, and introduce the notation of ∞-groupoids. The main references are [2, Section 1.1-1.2] and [4, Chapter 2.2].

Lecture 3. Anodyne maps and fibrations

Introduce the notation of (left, right, inner) anodyne maps and fibrations. Sketch the proof of the theorem that ∞-groupoids coincide with Kan complexes. The references are [4, Chapter2.1, 2.3] and [3, Section 1.3].

Lecture 4. Joining and slices, limits and colimits

Introduce joinings of simplicial sets and slices of simplicial sets and show that ∞-category is stable under these operations. Introduce initial objects and final objects, as well as limits and colimits in the setting of ∞-categories. The main references are [3, Section 1.4, 4.3]

Lecture 5. Cartesian and cocartesian fibrations, straightenings and unstraightenings

Introduce the (co)cartesian fibrations. Discuss the straightening and unstraightening theorem and formalism of adjoint functors. The main references are [4, Chapter 2.4, 3.2] and [3, Section 3.1, 3.3].

Lecture 6. ∞-category of presheaves and localizations

Study the ∞-category of presheaves, and the Yoneda embedding. Discuss localization of ∞-category. The main references are [4, Chapter 5.1, 5.2.7].

Lecture 7. Presentable ∞-categories I

Introduce the basic notation of presentable ∞-categories. The main references are [4, Chapter 5.5.1-2] and [2, Section 3]

Lecture 8. Presentable ∞-categories II

Study the ∞-category of presentable ∞-categories and truncations in presentable ∞-categories. The main references are [4, Chapter 5.5.3, 5.5.6].

Lecture 9. ∞-topoi I

Review the classical theory of topos, state Giraud's axioms and sketch the proof of it. Introduce some examples. The main references are [6, Lecture 10]

Lecture 10. ∞-topoi II

Introduce the notation of ∞-topoi and state Giraud's and Rezk's axioms and sketch the proof. Use Rezk's axioms to show the fundamental theorem of ∞-topoi. The main references are [4, Chapter 6.1]

Lecture 11. ∞-topoi III

Introduce the notation of ∞-category of ∞-topoi and state the alternative formulation of Rezk's axioms. Study the homotopy theory of ∞-topoi. The main references are [4, Chapter 6.3, 6.5]

Lecture 12. Stable ∞-categories I

Introduce the notation of Stable ∞-categories, show the homotopy category is a triangulated category and show they have the same t-structure. The main references are [5, Chapter 1.1, 1.2]

Lecture 13. Stable ∞-categories II

Give a quick introduction to the most important stable ∞-category–the category of spectra. The main references are [5, Chapter 1.4]

References

[1] D. Cisinski, *Higher categories and homotopical algebra*, Cambridge Studies in Advanced Mathematics, 180.
[2] M. Groth, *A short course on ∞-categories*, avaiable online.
[3] M. Land, *Introduction to infinity-categories*, Compact Textbooks in Mathematics.
[4] J. Lurie, *Higher topos theory*, avaiable online.
[5] J. Lurie, *Higher algebra*, avaiable online.
[6] J. Lurie, *Categorical logic*, lecture notes avaiable online.

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